

CBSE Board
Class XII Mathematics
Sample Paper 1

Time: 3 hrs

Total Marks: 100

General Instructions:

1. All the questions are **compulsory**.
2. The question paper consists of **37** questions divided into **three parts** A, B, and C.
3. **Part A** comprises of **20** questions of **1 mark** each. **Part B** comprises of **11** questions of **4 marks** each. **Part C** comprises of **6** questions of **6 marks** each.
4. There is no overall choice. However, an internal choice has been provided in **three questions of 4 marks** each, **four questions of 6 marks** each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculator is **not** permitted.

Section A

Q 1 – Q 20 are multiple choice type questions. Select the correct option.

1. Distance of a point P(a, b, c) from the z-axis is
 - A. 0
 - B. $\sqrt{a^2 + b^2}$
 - C. $\sqrt{b^2 + c^2}$
 - D. $\sqrt{a^2 + c^2}$
2. Two dice are thrown simultaneously. The probability of getting a pair of ones is
 - A. $\frac{1}{3}$
 - B. $\frac{1}{6}$
 - C. $\frac{1}{18}$
 - D. $\frac{1}{36}$
3. If $A = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find $(A + 2B)'$.
 - A. $\begin{bmatrix} 4 & 1 \\ 5 & -6 \end{bmatrix}$



- B. $\begin{bmatrix} -4 & 5 \\ -1 & 6 \end{bmatrix}$
- C. $\begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$
- D. $\begin{bmatrix} -4 & 1 \\ 5 & -6 \end{bmatrix}$

4. Angle between the two diagonals of a cube is

- A. 30°
- B. 45°
- C. $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$
- D. $\cos^{-1}\left(\frac{1}{3}\right)$

5. Find the projection of $\vec{a} = \hat{i} - 3\hat{k}$ on $\vec{b} = 3\hat{i} + \hat{j} - 4\hat{k}$.

- A. $\frac{15}{\sqrt{26}}$
- B. $\frac{\sqrt{15}}{26}$
- C. $\frac{\sqrt{26}}{15}$
- D. $\frac{26}{\sqrt{15}}$

6. Find the principal values of $\tan^{-1}(-1)$.

- A. $\frac{\pi}{4}$
- B. $\frac{\pi}{2}$
- C. $\frac{3\pi}{4}$
- D. π

7. Two cards are drawn from a well shuffled deck of 52 playing cards with replacement. The probability that both cards are queen is

- A. $\frac{1}{13} + \frac{1}{13}$
- B. $\frac{1}{13} \times \frac{1}{13}$



C. $\frac{1}{13} + \frac{1}{17}$

D. $\frac{1}{13} \times \frac{1}{17}$

8. The minimum value of $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 + \sin \theta & 1 \\ 1 + \cos \theta & 1 & 1 \end{vmatrix}$ (θ is real) is:

A. $-\frac{1}{2}$

B. $\frac{1}{2}$

C. -1

D. 1

9. Solve the system of inequations: $2x + y > 1$ and $2x - y \geq -3$ for x .

A. $x < \frac{1}{2}$

B. $x \leq -\frac{1}{2}$

C. $x > \frac{1}{2}$

D. $x \geq -\frac{1}{2}$

10. Integration of $\cos^{-1}(\sin x)$ is

A. $(\pi - x)$

B. $x(\pi - x)$

C. $\frac{x}{2}(\pi - x)$

D. $2x(\pi - x)$

11. The function $f(x) = x^9 + 3x^7 + 64$ is increasing on

A. Set of all real numbers

B. $(0, \infty)$

C. $(-\infty, 0)$

D. $\mathbb{R} - \{1\}$



12. If $A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$, $n \in \mathbb{N}$, then A^{4n} equals

A. $\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$

B. $\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

13. If $\vec{a} = 2\hat{i} - 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 4\hat{j} - 2\hat{k}$, then $\vec{a} \times \vec{b}$ is

A. $10\hat{i} + 3\hat{j} + 11\hat{k}$

B. $2\hat{i} + 5\hat{j} + 5\hat{k}$

C. $10\hat{i} - 3\hat{j} + 11\hat{k}$

D. $2\hat{i} - 5\hat{j} + 11\hat{k}$

14. If $a*b = a^2 + b^2$, then the value of $(4*5) * 3$ is

A. 60

B. 50

C. 1690

D. 1936

15. The value of b for which the function $f(x) = \begin{cases} 5x - 4 & , 0 < x \leq 1 \\ 4x^2 + 3bx & , 1 < x < 2 \end{cases}$ is continuous at every point of its domain is

A. $-\frac{5}{3}$

B. $-\frac{13}{3}$

C. -1

D. 0

16. Value of the integral $\int \tan^{-1} \left(\frac{\sin 2x}{1 + \cos 2x} \right) dx$ is

A. $\frac{x^2}{2} + c$

B. $x + c$

C. $\tan x + c$

D. 1



17. Area bounded by the curve $y = \cos x$ between $x = 0$ and $x = 2\pi$ is
- A. 0
 - B. 1 square unit
 - C. 2 square units
 - D. 4 square units
18. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then the value(s) of x is/are
- A. 3
 - B. ± 6
 - C. $3\sqrt{2}$
 - D. $\sqrt{2}2$
19. If $\int_0^{\alpha} \frac{1}{1+4x^2} dx = \frac{\pi}{8}$, the value of α is
- A. $\frac{1}{2} \tan \frac{\pi}{8}$
 - B. $\frac{1}{2}$
 - C. 1
 - D. $\frac{1}{2\sqrt{2}}$
20. Find integrating factor of the differential equation $(x \log x) \frac{dy}{dx} + y = 2 \log x$.
- A. x
 - B. $x \log x$
 - C. $\log x$
 - D. e^x



Section B

21. If θ is the angle between two vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$ then find $\sin \theta$.

22. Discuss the continuity of the function $f(x)$ at $x = 1$.

$$\text{Given : } f(x) = \begin{cases} \frac{3}{2} - x, & \frac{1}{2} \leq x \leq 1 \\ \frac{3}{2}, & x = 1 \\ \frac{3}{2} + x, & 1 < x \leq 2 \end{cases}$$

23. Solve the differential equation $(x^2 - y^2)dx + 2xy dy = 0$. Given that $y = 1$ when $x = 1$.

24. Evaluate: $\int_{-1}^2 (7x - 5) dx$, as a limit of sums.

OR

$$\text{Evaluate: } \int_2^3 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{5-x}} dx$$

25. A company has two plants of manufacturing scooters. Plant I manufactures 70% of the scooters and plant II manufactures 30%. At plant I, 30% of the scooters are rated of standard quality and at plant II, 90% of the scooters are rated of standard quality. A scooter is chosen at random and is found to be of standard quality. Find the probability that it is manufactured by plant II.

26. Show that the function $f: \mathbb{N} \rightarrow \mathbb{N}$ defined by $f(n) = n - (-1)^n$ for all $(n \in \mathbb{N})$, is a bijection.

OR

Show that relation R defined by $(a, b) R (c, d) \Rightarrow a + d = b + c$ on the set $\mathbb{N} \times \mathbb{N}$ is an equivalence relation.

27. Find the equation of the plane passing through the line of intersection of the planes $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ and $\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$ and parallel to x -axis.

28. Solve the equation: $\tan^{-1} \frac{x+1}{x-1} + \tan^{-1} \frac{x-1}{x} = \tan^{-1} (-7)$

29. Show that
$$\begin{vmatrix} 3p & -p+q & -p+r \\ -q+p & 3q & -q+r \\ -r+p & -r+q & 3r \end{vmatrix} = 3(p+q+r)(pq+qr+rp).$$



30. Solve the integral $\int x (\log x)^2 dx$

OR

Solve the integral $\int \frac{\cos x}{(1 - \sin x)(2 - \sin x)} dx$.

31. Find $\frac{d^2 y}{dx^2}$ if $x = a \left(\cos t + \log \tan \frac{t}{2} \right)$ and $y = a \sin t$.

Section C

32. Using integration, find the area of the circle $x^2 + y^2 = 16$ which is exterior to the parabola $y^2 = 6x$.

33. Find the angle between the lines whose direction cosines are given by the equations:
 $3l + m + 5n = 0$; $6mn - 2nl + 5lm = 0$.

OR

Find the distance of the point $(-1, -5, -10)$ from the point of intersection of the line and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$.

34. A window is in the form of a rectangle above which there is a semi-circle. If the perimeter of the window is p cm, show that the window will allow the maximum possible light only when the radius of the semi-circle is $\frac{p}{\pi + 4}$ cm.

OR

Show that the surface area of a closed cuboid with a square base and given volume is the least when it is a cube.

35. Find the inverse of the following matrix if exists, using elementary row transformation.

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$$

36. A factory produce bolts and nuts with the help of two machines A and B. It takes 3 hours on machine A and 1 hour on machine B to produce a pack of nuts. Similarly, a pack of bolts is produced by machines A and B working 1 hour and 3 hours respectively. The factory earns a profit of Rs. 19.60 per package on bolts and Rs. 9.00 per package on nuts. How many packages of each should be produced each day so as to maximize the profit, if the machines are operated at the most for 12 hours a day?

OR



One kind of chocolate requires 16g of flour and 3g of fat, and another kind 8g of flour and 6g of fat. Find the maximum number of chocolates which can be made from 0.4kg of flour and 0.12kg of fat assuming that there is no shortage of the other ingredients used in making the chocolates.

37. A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X ? Find mean and variance.

OR

An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of accidents are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?



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Section A

1. Correct option: B

Explanation:-

Coordinates of a point on the z-axis are $(0, 0, c)$

Therefore, we have

Distance of a point $P(a, b, c)$ from the z-axis

$$\begin{aligned} &= \sqrt{a^2 + b^2 + (c - c)^2} \\ &= \sqrt{a^2 + b^2} \end{aligned}$$

2. Correct option: D

Explanation:-

Probability of getting a one = $\frac{1}{6}$

Required probability = $P(\text{one on the first die}) \times P(\text{one on the second die})$

$$= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

3. Correct option: C

Explanation:-

$$A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$$

$$(A + 2B) = \begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}$$

$$(A + 2B)' = \begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$$



4. **Correct option: D**

Explanation:-

Angle between the two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$.

5. **Correct option: A**

Explanation:-

Projection of a vector \vec{a} on another vector \vec{b} is $\vec{a} \cdot \frac{\vec{b}}{|\vec{b}|}$

$$= (\hat{i} - 3\hat{k}) \cdot \frac{(3\hat{i} + \hat{j} - 4\hat{k})}{\sqrt{9 + 1 + 16}} = \frac{3 + 12}{\sqrt{26}} = \frac{15}{\sqrt{26}}$$

6. **Correct option: C**

Explanation:-

Let $x = \tan^{-1}(-1)$

$$\tan x = -1$$

$$\Rightarrow \tan x = -\tan \frac{\pi}{4}$$

$$\Rightarrow \tan x = \tan \left(\pi - \frac{\pi}{4} \right) \dots [\because \tan(\pi - \theta) = -\tan \theta]$$

$$\Rightarrow \tan x = \tan \frac{3\pi}{4}$$

$$\Rightarrow x = \frac{3\pi}{4}$$

7. **Correct option: B**

Explanation:-

Two cards are drawn from 52 cards

Let E_1 and E_2 be the events of getting queen in the first and second draws respectively.

$$\therefore P(E_1 \cap E_2) = \frac{4}{52} \times \frac{4}{52} = \frac{1}{13} \times \frac{1}{13}$$

8. **Correct option: A**

Explanation:-

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 + \sin \theta & 1 \\ 1 + \cos \theta & 1 & 1 \end{vmatrix}$$

$$R_1 \rightarrow R_1 - R_2$$

$$\Delta = \begin{vmatrix} 0 & -\sin \theta & 0 \\ 1 & 1 + \sin \theta & 1 \\ 1 + \cos \theta & 1 & 1 \end{vmatrix}$$

$$\Delta = \sin \theta (1 - 1 - \cos \theta) = -\sin \theta \cos \theta = -\frac{\sin 2\theta}{2}$$

We know that $-1 \leq \sin x \leq 1$

$$\text{Therefore, } -\frac{1}{2} \leq -\frac{\sin x}{2} \leq \frac{1}{2}$$

Hence, the minimum value of Δ is $-\frac{1}{2}$.

9. Correct option: D

Explanation:-

Given: $2x + y > 1$ and $2x - y \geq -3$

$$\Rightarrow y > 1 - 2x \text{ and } 2x + 3 \geq y$$

$$\Rightarrow 1 - 2x \leq 2x + 3$$

$$\Rightarrow x \geq -\frac{1}{2}$$

10. Correct option: C

Explanation:-

$$\text{Let } I = \int \cos^{-1}(\sin x) dx$$

$$\text{We know that } \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

$$\Rightarrow \cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$$

$$\therefore I = \int \left[\frac{\pi}{2} - \sin^{-1}(\sin x) \right] dx$$

$$\therefore I = \int \frac{\pi}{2} dx - \int x dx$$

$$\therefore I = \frac{\pi}{2} x - \frac{x^2}{2} = \frac{x}{2} (\pi - x)$$

11. Correct option: A

Explanation:-

$$f(x) = x^9 + 3x^7 + 64$$

$$\Rightarrow f'(x) = 9x^8 + 21x^6 = 3x^6(3x^2 + 7)$$

$$\text{As } 3x^6(3x^2 + 7) > 0 \quad \forall x \in \mathbb{R}$$

Hence, f is increasing on \mathbb{R} .

12. Correct option: C

Explanation:-

$$\text{Given: } A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} = i \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^4 = i^4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^{4n} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

13. Correct option: A

Explanation:-

$$\text{Given: } \vec{a} = 2\hat{i} - 3\hat{j} - \hat{k} \text{ and } \vec{b} = \hat{i} + 4\hat{j} - 2\hat{k},$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & -1 \\ 1 & 4 & -2 \end{vmatrix}$$

$$\vec{a} \times \vec{b} = 10\hat{i} + 3\hat{j} + 11\hat{k}$$

14. Correct option: C

Explanation:-

$$\text{Given: } a * b = a^2 + b^2$$

$$\text{So, } 4 * 5 = 4^2 + 5^2$$

$$(4 * 5) * 3 = (4 * 5)^2 + 3^2$$

$$= (4^2 + 5^2)^2 + 3^2$$

$$= 41^2 + 3^2 = 1690$$

15. Correct option: C

Explanation:-

Function f is continuous at every point of its domain, so f is continuous at 1.

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x)$$

$$\Rightarrow \lim_{x \rightarrow 1} 5x - 4 = \lim_{x \rightarrow 1} 4x^2 + 3bx$$

$$\Rightarrow 5 - 4 = 4 + 3b$$

$$\Rightarrow 1 = 4 + 3b$$

$$\Rightarrow b = -1$$



16. Correct option: A

Explanation:-

$$\text{Let } I = \int \tan^{-1} \left(\frac{\sin 2x}{1 + \cos 2x} \right) dx$$

$$I = \int \tan^{-1} \left(\frac{2 \sin x \cos x}{1 + 2 \cos^2 x - 1} \right) dx$$

$$= \int \tan^{-1} \left(\frac{\sin x}{\cos x} \right) dx$$

$$= \int \tan^{-1} (\tan x) dx$$

$$= \int x dx$$

$$= \frac{x^2}{2} + c$$

17. Correct option: D

Explanation:-

The required area is given by

$$A = \int_0^{2\pi} \cos x dx$$

$$= \int_0^{\frac{\pi}{2}} \cos x dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos x dx + \int_{\frac{3\pi}{2}}^{2\pi} \cos x dx$$

$$= \left[\sin x \right]_0^{\frac{\pi}{2}} + \left[\sin x \right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} + \left[\sin x \right]_{\frac{3\pi}{2}}^{2\pi}$$

$$= 1 + 2 + 1 \dots (\text{Since area can't be -ve})$$

$$= 4 \text{ sq. units}$$

18. Correct option: B

Explanation:-

Given:

$$\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$$

$$\Rightarrow 2x^2 - 40 = 18 + 14$$

$$\Rightarrow 2x^2 = 32 + 40$$

$$\Rightarrow 2x^2 = 72$$

$$\Rightarrow x^2 = 36$$

$$\Rightarrow x = \pm 6$$

19. Correct option: B

Explanation:-

Given:

$$\int_0^{\alpha} \frac{1}{1+4x^2} dx = \frac{\pi}{8}$$

$$\Rightarrow \frac{1}{2} [\tan^{-1}(2x)]_0^{\alpha} = \frac{\pi}{8}$$

$$\Rightarrow \tan^{-1}(2\alpha) = \frac{\pi}{4}$$

$$\Rightarrow 2\alpha = \tan^{-1} \frac{\pi}{4} \Rightarrow \alpha = \frac{1}{2}$$

20. Correct option: C

Explanation:-

Given differential equation is:

$$(x \log x) \frac{dy}{dx} + y = 2 \log x$$

$$\Rightarrow \frac{dy}{dx} + \frac{1}{x \log x} y = \frac{2 \log x}{x \log x}$$

$$\Rightarrow \frac{dy}{dx} + \frac{1}{x \log x} y = \frac{2}{x}$$

Comparing with $\frac{dy}{dx} + Py = Q$

$$\Rightarrow P = \frac{1}{x \log x}, Q = \frac{2}{x}$$

$$\text{I.F.} = e^{\int P dx} = e^{\int \frac{1}{x \log x} dx} = e^{\log(\log x)} = \log x$$



Section B

21. Given two vectors are $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$.

$$\sin \theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}| |\vec{b}|} \dots\dots\dots (i)$$

To find $\vec{a} \times \vec{b}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 3 & -2 & 1 \end{vmatrix}$$

$$\Rightarrow \vec{a} \times \vec{b} = [-2 - (-6)]\hat{i} - (1 - 9)\hat{j} + [-2 - (-6)]\hat{k}$$

$$\Rightarrow \vec{a} \times \vec{b} = 4\hat{i} + 8\hat{j} + 4\hat{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{4^2 + 8^2 + 4^2} = \sqrt{96} = 4\sqrt{6} \dots\dots\dots (i)$$

$$|\vec{a}| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{14} \dots\dots\dots (ii) \text{ and}$$

$$|\vec{b}| = \sqrt{3^2 + (-2)^2 + 1^2} = \sqrt{14} \dots\dots\dots (iii)$$

$$\Rightarrow \sin \theta = \frac{4\sqrt{6}}{\sqrt{14}\sqrt{14}} \dots\dots\dots \text{From (i), (ii) and (iii)}$$

$$\Rightarrow \sin \theta = \frac{4\sqrt{6}}{14} = \frac{2\sqrt{6}}{7}$$

22. The function $f(x)$ is continuous at $x = a$ if $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) = f(a)$

We need to check the continuity of $f(x)$ at $x = 1$

So consider,

$$\text{L.H.L.} = \lim_{x \rightarrow 1^-} f(x)$$

$$= \lim_{x \rightarrow 1} \left(\frac{3}{2} - x \right)$$

$$= \frac{3}{2} - 1 = \frac{1}{2}$$

And,

$$\text{R.H.L.} = \lim_{x \rightarrow 1^+} f(x)$$

$$= \lim_{x \rightarrow 1} \left(\frac{3}{2} + x \right)$$

$$= \frac{3}{2} + 1 = \frac{5}{2}$$

$$\Rightarrow \text{L.H.L.} \neq \text{R.H.L.}$$

Therefore, function $f(x)$ is not continuous at $x = 1$.



23. Consider $(x^2 - y^2)dx + 2xy dy = 0$ (i)

Given equation is homogeneous, then take $y = vx$

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substituting the values in (i), we get

$$(x^2 - v^2 x^2) + 2x^2 v \cdot \frac{dy}{dx} = 0$$

$$\Rightarrow x^2 (1 - v^2) + 2x^2 v \left(v + x \frac{dv}{dx} \right) = 0$$

$$\Rightarrow x^2 (1 - v^2) + 2x^2 v^2 + 2x^3 v \cdot \frac{dv}{dx} = 0$$

$$\Rightarrow x^2 (1 - v^2 + 2v^2) + 2x^3 v \frac{dv}{dx} = 0$$

$$\Rightarrow x^2 (1 + v^2) + 2x^3 v \frac{dv}{dx} = 0$$

$$\Rightarrow \int \frac{v}{1 + v^2} dv + \int \frac{x^2}{2x^3} dx = 0$$

$$\Rightarrow \frac{1}{2} \log(1 + v^2) + \frac{1}{2} \log x = \log C$$

$$\Rightarrow \left(1 + \frac{y^2}{x^2} \right) x = C_1$$

$$\Rightarrow (x^2 + y^2) = C_1 x$$

Now $y = 1$ when $x = 1$

So, $2 = C_1$

\therefore Solution is $x^2 + y^2 = 2x$

24. Let $I = \int_{-1}^2 (7x - 5) dx$

Take $a = -1$, $b = 2$; $h = \frac{2+1}{n} \Rightarrow nh = 3, f(x) = 7x - 5$

$$\therefore I = \lim_{h \rightarrow 0} h \left[f(-1) + f(-1+h) + f(-1+2h) + \dots + f(-1 + \overline{n-1}h) \right] \dots (i)$$

$$f(-1) = -7 - 5 = -12; f(-1+h) = 7(-1+h) - 5 = 7h - 12$$

$$f(-1 + \overline{n-1}h) = 7(-1 + \overline{n-1}h) - 5 = 7(n-1)h - 12$$

Substituting in (i)

$$\int_{-1}^2 (7x - 5) dx = \lim_{h \rightarrow 0} h \left[(-12) + (7h - 12) + (14h - 12) + \dots + \{7(n-1)h - 12\} \right]$$

$$\begin{aligned}
&= \lim_{h \rightarrow 0} h \left[7h(1 + 2 + \dots + n - 1) - 12n \right] = \lim_{h \rightarrow 0} h \left[7h \frac{(n-1)n}{2} - 12n \right] \\
&= \lim_{h \rightarrow 0} \left[\frac{7}{2}(nh)(nh - h) - 12nh \right] = \lim_{h \rightarrow 0} \left[\frac{7}{2}(3)(3 - h) - 36 \right] \\
&= \frac{7}{2} \times 9 - 36 = \frac{63}{2} - 36 = -\frac{9}{2}
\end{aligned}$$

OR

$$\text{Let } I = \int_2^3 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{5-x}} dx \quad \dots (i)$$

Using $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$, we get

$$I = \int_2^3 \frac{\sqrt{2+3-x}}{\sqrt{2+3-x} + \sqrt{2+3-5+x}} dx$$

$$\Rightarrow I = \int_2^3 \frac{\sqrt{5-x}}{\sqrt{5-x} + \sqrt{x}} dx \quad \dots (ii)$$

Adding (i) and (ii), we get

$$2I = \int_2^3 1 \cdot dx$$

$$= x \Big|_2^3 = 1$$

$$\Rightarrow I = \frac{1}{2}$$

25. Let E: Standard quality

$$P(E/I) = \frac{30}{100}; P(E/II) = \frac{90}{100}$$

$$P(II/E) = \frac{P(II) \cdot P(E/II)}{P(I) \cdot P(E/I) + P(II) \cdot P(E/II)}$$

$$\begin{aligned}
&= \frac{\frac{30}{100} \times \frac{90}{100}}{\frac{70}{100} \times \frac{30}{100} + \frac{30}{100} \times \frac{90}{100}} \\
&= \frac{9}{16}
\end{aligned}$$

26. $f(n) = n - (-1)^n$

$$f(n) = n - 1, n \text{ is even}$$

$$= n + 1, n \text{ is odd}$$

Injectivity:

Let n, m be any two even/odd natural numbers.

$$\text{Then, } f(n) = f(m)$$

$$\Rightarrow n - 1 = m - 1$$



$$\Rightarrow n = m$$

Thus, in both cases $f(n) = f(m) \Rightarrow n = m$

If n is even and m is odd, then $n \neq m$. Also $f(n)$ is odd and $f(m)$ is even.

So, $f(n) \neq f(m)$.

Thus $n \neq m \Rightarrow f(n) \neq f(m)$

So, f is an injective map.

Surjectivity:

Let n be an arbitrary natural number.

If n is an odd natural number, then there exists an even natural number $n + 1$ such that

$$f(n + 1) = n + 1 - 1 = n$$

If n is an even natural number, then there exists an odd natural number $n + 1$ such that

$$f(n + 1) = n + 1 + 1 = n + 2$$

Thus, every $n \in \mathbb{N}$ has its pre-image in \mathbb{N} .

So, $f: \mathbb{N} \rightarrow \mathbb{N}$ is a surjection.

Hence, $f: \mathbb{N} \rightarrow \mathbb{N}$ is a bijection.

OR

$$(a, b) R (c, d) = a + d = b + c$$

For reflexive: $(a, b) R (a, b) \Rightarrow a + b = b + a$, true in \mathbb{N}

For symmetric: $(a, b) R (c, d) \Rightarrow a + d = b + c \Rightarrow c + b = d + a$

$\Rightarrow (c, d) R (a, b)$, for $(a, b), (c, d) \in \mathbb{N} \times \mathbb{N}$

For transitive: Let $(a, b), (c, d), (e, f) \in \mathbb{N} \times \mathbb{N}$

$(a, b) R (c, d)$ and $(c, d) R (e, f)$

$$(a, b) R (c, d) \Rightarrow a + d = b + c \quad \dots\dots\dots(i)$$

$$(c, d) R (e, f) \Rightarrow c + f = d + e \quad \dots\dots\dots(ii)$$

Adding (i) and (ii), we get

$$a + d + c + f = b + c + d + e \Rightarrow a + f = b + e \Rightarrow (a, b) R (e, f)$$

Hence, R is transitive

Hence, the relation R is an equivalence relation.

27. The given planes are

$$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1 \Rightarrow \vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) - 1 = 0 \dots (i)$$

$$\text{And, } \vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0 \dots (ii)$$

The equation of plane passing through the line of intersection of these planes is

$$\left[\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) - 1 \right] + \lambda \left[\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 \right] = 0$$

$$\Rightarrow \vec{r} \cdot \left[(2\lambda + 1)\hat{i} + (3\lambda + 1)\hat{j} + (1 - \lambda)\hat{k} \right] + (4\lambda - 1) = 0 \dots (iii)$$

Its direction ratios are $(2\lambda + 1)$, $(3\lambda + 1)$ and $(1 - \lambda)$.

The required plane is parallel to x-axis. So, its normal will be perpendicular to x-axis.

We know that, the direction ratios of x-axis are 1, 0 and 0

$$\Rightarrow 1 \cdot (2\lambda + 1) + 0(3\lambda + 1) + 0(1 - \lambda) = 0$$

$$\Rightarrow 2\lambda + 1 = 0$$

$$\Rightarrow \lambda = -\frac{1}{2}$$

Substituting $\lambda = -\frac{1}{2}$ in equation (iii), we get

$$\Rightarrow \vec{r} \cdot \left[-\frac{1}{2}\hat{j} + \frac{3}{2}\hat{k} \right] + (-3) = 0$$

$$\Rightarrow \vec{r} \cdot (\hat{j} - 3\hat{k}) + 6 = 0$$

Therefore, its Cartesian equation is $y - 3z + 6 = 0$

This is the equation of the required plane.

28. The given equation is $\tan^{-1} \frac{x+1}{x-1} + \tan^{-1} \frac{x-1}{x} = \tan^{-1}(-7)$

$$\Rightarrow \tan^{-1} \left(\frac{\frac{x+1}{x-1} + \frac{x-1}{x}}{1 - \frac{x+1}{x-1} \times \frac{x-1}{x}} \right) = \tan^{-1}(-7)$$

$$\Rightarrow \tan^{-1} \left(\frac{x^2 + x + x^2 - 2x + 1}{x^2 - x - x^2 + 1} \right) = \tan^{-1}(-7)$$

$$\Rightarrow \tan^{-1} \left(\frac{2x^2 - x + 1}{-x + 1} \right) = \tan^{-1}(-7)$$

$$\Rightarrow \frac{2x^2 - x + 1}{-x + 1} = -7$$

$$\Rightarrow 2x^2 - 8x + 8 = 0$$

$$\Rightarrow x^2 - 4x + 4 = 0$$

$$\Rightarrow (x - 2)^2 = 0$$

$$\Rightarrow x = 2$$

But $x = 2$ does not satisfy the equation $\tan^{-1} \frac{x+1}{x-1} + \tan^{-1} \frac{x-1}{x} = \tan^{-1}(-7)$

Hence, no solution exist.

$$\text{29. LHS} = \begin{vmatrix} 3p & -p+q & -p+r \\ -q+p & 3q & -q+r \\ -r+p & -r+q & 3r \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$, we get

$$\text{LHS} = \begin{vmatrix} p+q+r & -p+q & -p+r \\ p+q+r & 3q & -q+r \\ p+q+r & -r+q & 3r \end{vmatrix} = (p+q+r) \begin{vmatrix} 1 & -p+q & -p+r \\ 1 & 3q & -q+r \\ 1 & -r+q & 3r \end{vmatrix}$$

Applying $R_1 \rightarrow R_1 - R_2$

$$\text{LHS} = (p+q+r) \begin{vmatrix} 0 & -p-2q & -p+q \\ 1 & 3q & -q+r \\ 1 & -r+q & 3r \end{vmatrix}$$

Applying $R_2 \rightarrow R_2 - R_3$

$$\text{LHS} = (p+q+r) \begin{vmatrix} 0 & -p-2q & -p+q \\ 0 & r+2q & -q-2r \\ 1 & -r+q & 3r \end{vmatrix}$$

Expanding this determinant along C_1 , we get

$$\begin{aligned} \text{LHS} &= (p+q+r)[(-p-2q)(-q-2r) - (-p+q)(r+2q)] \\ &= (p+q+r)(pq + 2pr + 2q^2 + 4qr + pr + 2pq - qr - 2q^2) \\ &= (p+q+r)(3pq + 3qr + 3pr) \\ &= 3(p+q+r)(pq + qr + rp) = \text{RHS} \end{aligned}$$

$$\text{Hence, } \begin{vmatrix} 3p & -p+q & -p+r \\ -q+p & 3q & -q+r \\ -r+p & -r+q & 3r \end{vmatrix} = 3(p+q+r)(pq + qr + rp).$$

30. Taking $(\log x)^2$ as the first function and x as the second function for applying integrating by parts, we have

$$\begin{aligned} I &= (\log x)^2 \int x \, dx - \int \left\{ \left[\frac{d}{dx} (\log x)^2 \right] \int x \, dx \right\} dx \\ &= (\log x)^2 \left(\frac{x^2}{2} \right) - \int \left[\left(\frac{2 \log x}{x} \right) \left(\frac{x^2}{2} \right) \right] dx \\ &= (\log x)^2 \left(\frac{x^2}{2} \right) - \int (x \log x) \, dx \end{aligned}$$

Again applying By parts, we get

$$\begin{aligned}
 I &= (\log x)^2 \left(\frac{x^2}{2} \right) - \left\{ \log x \int x \, dx - \int \left[\left(\frac{d}{dx} \log x \right) \int x \, dx \right] dx \right\} \\
 &= (\log x)^2 \left(\frac{x^2}{2} \right) - \left\{ \log x \left(\frac{x^2}{2} \right) - \int \left[\left(\frac{1}{x} \right) \frac{x^2}{2} \right] dx \right\} \\
 &= (\log x)^2 \left(\frac{x^2}{2} \right) - (\log x) \left(\frac{x^2}{2} \right) + \int \frac{x}{2} \, dx \\
 &= (\log x) \left(\frac{x^2}{2} \right) (\log x - 1) + \frac{x^2}{4} + C
 \end{aligned}$$

$$\text{Hence, } \int x (\log x)^2 \, dx = (\log x) \left(\frac{x^2}{2} \right) (\log x - 1) + \frac{x^2}{4} + C.$$

OR

$$\text{Let } I = \int \frac{\cos x}{(1 - \sin x)(2 - \sin x)} \, dx$$

Take $\sin x = t \Rightarrow \cos x \, dx = dt$, we get

$$I = \int \frac{dt}{(1 - t)(2 - t)}$$

$$\text{Let } \frac{1}{(1 - t)(2 - t)} = \frac{A}{1 - t} + \frac{B}{2 - t}$$

$$\Rightarrow 1 = A(2 - t) + B(1 - t)$$

Take $t = 2$ and then $t = 1$, we get

$$B = -1 \text{ and } A = 1$$

$$\Rightarrow \frac{1}{(1 - t)(2 - t)} = \frac{1}{1 - t} - \frac{1}{2 - t}$$

Therefore, we have

$$\begin{aligned}
 I &= \int \left[\frac{1}{(1 - t)} - \frac{1}{(2 - t)} \right] dt \\
 &= -\log |1 - t| + \log |2 - t| + C \\
 &= \log \left| \frac{2 - t}{1 - t} \right| + C \\
 &= \log \left| \frac{2 - \sin x}{1 - \sin x} \right| + C
 \end{aligned}$$

31. Given: $x = a \left(\cos t + \log \tan \frac{t}{2} \right)$ and $y = a \sin t$

Differentiating x w.r.t. t , we get



$$\begin{aligned}
 \frac{dx}{dt} &= a \left[-\sin t + \cot \frac{t}{2} \times \frac{d}{dt} \left(\tan \frac{t}{2} \right) \right] \\
 \Rightarrow \frac{dx}{dt} &= a \left[-\sin t + \cot \frac{t}{2} \times \sec^2 \frac{t}{2} \times \frac{d}{dt} \left(\frac{t}{2} \right) \right] \\
 \Rightarrow \frac{dx}{dt} &= a \left[-\sin t + \cot \frac{t}{2} \times \sec^2 \frac{t}{2} \times \frac{1}{2} \right] \\
 \Rightarrow \frac{dx}{dt} &= a \left[-\sin t + \frac{1}{\sin t} \right] = a \left[\frac{1 - \sin^2 t}{\sin t} \right] \\
 \Rightarrow \frac{dx}{dt} &= a \left(\frac{\cos^2 t}{\sin t} \right) \dots (i)
 \end{aligned}$$

Differentiating y w.r.t. t, we get

$$\frac{dy}{dt} = a \cos t \dots (ii)$$

From (i) and (ii), we have

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{a \cos t}{a \left(\frac{\cos^2 t}{\sin t} \right)} = \frac{\sin t}{\cos t} = \tan t$$

$$\Rightarrow \frac{dy}{dx} = \tan t \dots (iii)$$

$$\text{Now, } \frac{d^2 y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right)$$

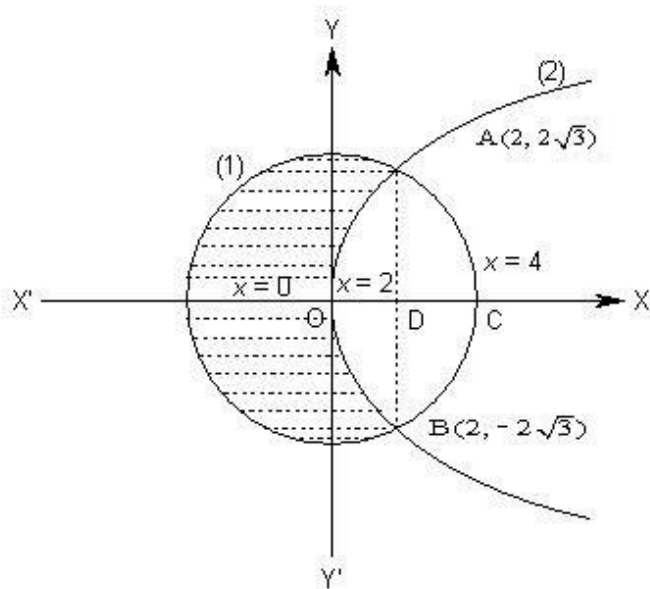
From (iii), we get

$$\begin{aligned}
 \frac{d^2 y}{dx^2} &= \frac{d}{dx} (\tan t) = \sec^2 t \cdot \frac{dt}{dx} = \sec^2 t \cdot \frac{\sin t}{a \cos^2 t} = \frac{\sin t}{a \cos^4 t} \\
 \Rightarrow \frac{d^2 y}{dx^2} &= \frac{\sec^4 t \cdot \sin t}{a}
 \end{aligned}$$

Section C

32. $x^2 + y^2 = 16$ (i)

$y^2 = 6x$ (ii)



Points of intersection of curve (i) and (ii) are obtained as follows:

$$x^2 + 6x - 16 = 0$$

$$\Rightarrow (x + 8)(x - 2) = 0$$

$$\Rightarrow x = 2 \quad (\because x \neq -8)$$

$$y^2 = 12$$

$$y = \pm 2\sqrt{3}$$

Therefore, points of intersection are A $(2, 2\sqrt{3})$ and B $(2, -2\sqrt{3})$

Also, C $(0, 4)$

$$\text{Area}(\text{OB CAO}) = 2 (\text{Area ODA} + \text{Area DCA})$$

$$= 2 \left[\int_0^2 \sqrt{6x} \, dx + \int_2^4 \sqrt{16 - x^2} \, dx \right]$$

$$= 2 \left[\sqrt{6} \cdot \left\{ \frac{2}{3} x^{3/2} \right\}_0^2 + \left\{ \frac{x\sqrt{16 - x^2}}{2} + \frac{16}{2} \sin^{-1} \frac{x}{4} \right\}_2^4 \right]$$

$$= 2 \left[\frac{2\sqrt{6}}{3} \cdot 2\sqrt{2} + \left\{ 0 + 8 \sin^{-1} 1 \right\} - \left\{ \frac{2 \cdot 2\sqrt{3}}{2} + 8 \sin^{-1} \frac{1}{2} \right\} \right]$$

$$= \frac{16\sqrt{3}}{3} + 16 \cdot \frac{\pi}{2} - \left(4\sqrt{3} + 16 \cdot \frac{\pi}{6} \right)$$

$$= \left(\frac{4\sqrt{3}}{3} + \frac{16}{3}\pi \right) \text{sq. units.}$$

$$\text{Required Area} = \text{Area of a circle} - \left(\frac{4\sqrt{3}}{3} + \frac{16}{3}\pi \right)$$

$$= 16\pi - \frac{4\sqrt{3}}{3} - \frac{16}{3}\pi$$

$$= \frac{32}{3}\pi - \frac{4\sqrt{3}}{3} = \frac{4}{3}(8\pi - \sqrt{3}) \text{sq. units}$$

33. $3l + m + 5n = 0 \dots (1)$

$$6mn - 2nl + 5lm = 0 \dots (2)$$

$$\text{From (1), } m = -(3l + 5n) \dots (3)$$

Substituting the value of m in (2)

$$\Rightarrow -6(3l + 5n)n - 2nl - 5l(3l + 5n) = 0$$

$$\Rightarrow -18ln - 30n^2 - 2nl - 15l^2 - 25nl = 0$$

$$\Rightarrow -30n^2 - 45nl - 15l^2 = 0$$

$$\Rightarrow 2n^2 + 3ln + l^2 = 0 \quad [\text{Divide by } (-15)]$$

$$\Rightarrow (2n + l)(n + l) = 0$$

$$\text{Either } 2n + l = 0 \text{ or } n + l = 0$$

$$(I) \text{ when } 2n + l = 0 \Rightarrow l = -2n$$

$$\text{From (3), } m = -(-6n + 5n) = n$$

$$(II) \text{ when } n + l = 0 \Rightarrow l = -n$$

$$\text{From (3), } m = -(-3n + 5n) = -2n$$

Direction ratios of two lines are

$$-2n, n, n \text{ and } -n, -2n, n$$

$$-2, 1, 1 \text{ and } 1, 2, -1$$

Angle between two lines is

$$\cos \theta = \left| \frac{(-2)1 + (1)(2) + (1)(-1)}{\sqrt{(-2)^2 + 1^2 + 1^2} \sqrt{1^2 + 2^2 + (-1)^2}} \right| = \frac{1}{6}$$

$$\text{Hence, } \theta = \cos^{-1} \left(\frac{1}{6} \right).$$



OR

Equation of line is $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$

$$\vec{r} = (2 + 3\lambda)\hat{i} + (-1 + 4\lambda)\hat{j} + (2 + 2\lambda)\hat{k}$$

This point lies on the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$

$$\Rightarrow [(2 + 3\lambda)\hat{i} + (-1 + 4\lambda)\hat{j} + (2 + 2\lambda)\hat{k}] \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$$

$$\Rightarrow (2 + 3\lambda) + (-1 + 4\lambda)(-1) + (2 + 2\lambda) = 5$$

$$\Rightarrow \lambda = 0$$

Coordinates are

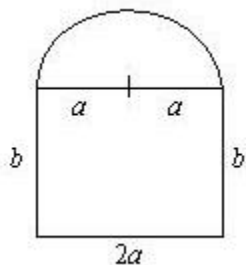
$$(2 + 3\lambda, -1 + 4\lambda, 2 + 2\lambda) = (2, -1, 2)$$

Distance between $(2, -1, 2)$ and $(-1, -5, -10)$

$$= \sqrt{(-1 - 2)^2 + (-5 + 1)^2 + (-10 - 2)^2}$$

$$= 13 \text{ units}$$

34. Let $2a$ cm be the length and b cm be the breadth of the rectangle. Then a cm is the radius of the semi-circle.



By hypothesis,

$$\text{perimeter} = 2a + b + b + \pi a$$

$$\Rightarrow p = 2a + b + b + \pi a$$

$$\Rightarrow 2b = p - (\pi + 2)a \dots (1)$$

Also A = Area of the window

$$= \frac{1}{2}\pi a^2 + 2a \times b$$

$$= \frac{1}{2}\pi a^2 + a \times [p - (\pi + 2)a] \dots [\text{By (1)}]$$

$$= pa - \frac{1}{2}(\pi + 4)a^2$$

$$\Rightarrow \frac{dA}{da} = p - (\pi + 4)a$$

$$\text{For max or min } \frac{dA}{da} = 0$$

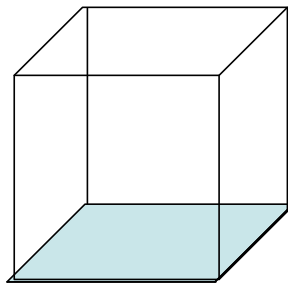
$$\Rightarrow p - (\pi + 4) a = 0$$

$$\Rightarrow a = \frac{p}{\pi + 4}.$$

$$\text{Also, } \frac{d^2 A}{da^2} = -(\pi + 4) < 0$$

\therefore The light will be maximum when the radius of the semi-circle is, $a = \frac{p}{\pi + 4}$.

OR



Let x be side of the square base and y be the height of the cuboid

$$\text{Volume (V)} = x \cdot x \cdot y = x^2 y \dots (i)$$

$$\text{Surface area (S)} = 2 (x \cdot x + x \cdot y + x \cdot y) = 2x^2 + 4xy = 2x^2 + 4x \frac{V}{x^2}$$

$$S = 2x^2 + \frac{4V}{x} \Rightarrow \frac{dS}{dx} = 4x - \frac{4V}{x^2}$$

$$\text{For minimum S, } \frac{dS}{dx} = 0 \Rightarrow 4x - \frac{4V}{x^2} = 0 \Rightarrow x^3 = V \Rightarrow x = \sqrt[3]{V}$$

$$\frac{d^2 S}{dx^2} = 4 + \frac{8V}{x^3} \Rightarrow \left. \frac{d^2 S}{dx^2} \right|_{x=\sqrt[3]{V}} = 4 + \frac{8V}{V} > 0$$

\therefore For $x = \sqrt[3]{V}$, surface area is minimum

$$\Rightarrow x^3 = V \Rightarrow x^3 = x^2 y \text{ [From (i)]} \Rightarrow x = y \Rightarrow \text{cuboid is a cube}$$

35. Consider, $A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$

We write $A = IA$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

[By performing $R_2 \rightarrow R_2 + 3R_1$]

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 0 & 9 & -11 & 0 \\ 0 & -1 & 4 & 1 \end{array} \right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ -2 & 0 & 1 & 1 \end{array} \right] A \quad [\text{By performing } R_3 \rightarrow R_3 - 2R_1]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 10 & 0 \\ 0 & 9 & -11 & 0 \\ 0 & -1 & 4 & 1 \end{array} \right] = \left[\begin{array}{ccc|c} -5 & 0 & 3 & 0 \\ 3 & 1 & 0 & 0 \\ -2 & 0 & 1 & 1 \end{array} \right] A \quad [\text{By performing } R_1 \rightarrow R_1 + 3R_3]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 10 & 0 \\ 0 & 1 & -\frac{11}{9} & 0 \\ 0 & -1 & 4 & 1 \end{array} \right] = \left[\begin{array}{ccc|c} -5 & 0 & 3 & 0 \\ \frac{1}{3} & \frac{1}{9} & 0 & 0 \\ -2 & 0 & 1 & 1 \end{array} \right] A \quad [\text{By performing } R_2 \rightarrow \frac{1}{9} R_2]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 10 & 0 \\ 0 & 1 & -\frac{11}{9} & 0 \\ 0 & 0 & \frac{25}{9} & 1 \end{array} \right] = \left[\begin{array}{ccc|c} -5 & 0 & 3 & 0 \\ \frac{1}{3} & \frac{1}{9} & 0 & 0 \\ -\frac{5}{3} & \frac{1}{9} & 1 & 1 \end{array} \right] A \quad [\text{By performing } R_3 \rightarrow R_3 + R_2]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 10 & 0 \\ 0 & 1 & -\frac{11}{9} & 0 \\ 0 & 0 & 1 & 1 \end{array} \right] = \left[\begin{array}{ccc|c} -5 & 0 & 3 & 0 \\ \frac{1}{3} & \frac{1}{9} & 0 & 0 \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} & 1 \end{array} \right] A \quad [\text{By performing } R_3 \rightarrow \frac{9}{25} R_3]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 10 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right] = \left[\begin{array}{ccc|c} -5 & 0 & 3 & 0 \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} & 0 \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} & 1 \end{array} \right] A \quad [\text{By performing } R_2 \rightarrow R_2 + \frac{11}{9} R_3]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right] = \left[\begin{array}{ccc|c} 1 & -\frac{2}{5} & -\frac{3}{5} & 0 \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} & 0 \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} & 1 \end{array} \right] A \quad [\text{By performing } R_1 \rightarrow R_1 - 10R_3]$$

Hence, we obtain

$\Rightarrow B$ is inverse of A by definition.

$$\text{Hence, } A^{-1} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$$

36. Let the number of packages of bolts = x and,

Number of packages of nuts = y

To maximize profit $Z = 19.6x + 9y$

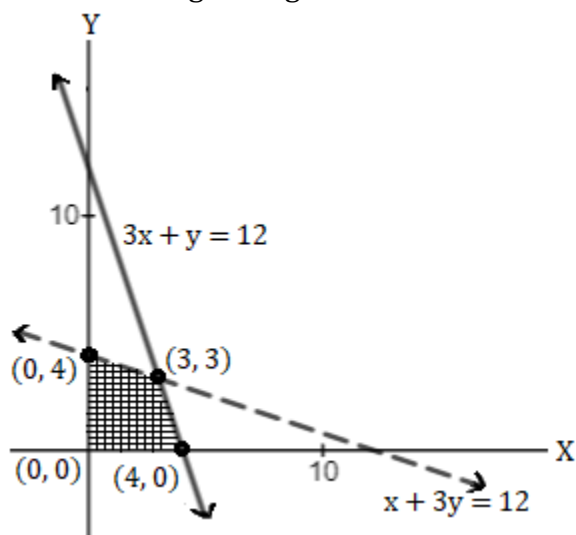
As per the question,

$3y + x \leq 12$ and,

$y + 3x \leq 12$

$x \geq 0, y \geq 0$

The feasible region is given below:



Here, the points lying on $x + 3y = 12$ are $(0, 4)$ and $(12, 0)$

Also, the points lying on $3x + y = 12$ are $(0, 12)$ and $(4, 0)$

The feasible region contains $(0, 0)$ as it satisfies both the inequality.

The point of intersection of both the equation is $(3, 3)$

We have the corner points of the feasible region are $(0, 0)$, $(4, 0)$, $(0, 4)$ and $(3, 3)$

$Z = 19.6x + 9y$

At $(0, 0)$, $Z = 19.6 \times 0 + 9 \times 0 = 0$

At $(4, 0)$, $Z = 19.6 \times 4 + 9 \times 0 = 78.4$

At $(0, 4)$, $Z = 19.6 \times 0 + 9 \times 4 = 36$

At $(3, 3)$, $Z = 19.6 \times 3 + 9 \times 3 = 58.8 + 27 = 85.8$

Therefore, maximum profit = Rs. 85.8 at $(3, 3)$

Hence, the profit is maximum when the number of packets of bolts and that of nuts is 3 each.

OR

Let there be x chocolates of one kind and y chocolates of other kind.

Therefore, $x \geq 0, y \geq 0$

The given information is as follows:

	Flour(g)	Fat (g)
Chocolates of 1 st kind, x	16	3
Chocolates of 2 nd kind, y	8	6
Availability	400	120

$$16x + 8y \leq 400 \Rightarrow 2x + y \leq 50$$

$$3x + 6y \leq 120 \Rightarrow x + 2y \leq 40$$

Total number of chocolates is $Z = x + y$

The mathematical formation of the given problem is

Maximize $Z = x + y \dots$ (i)

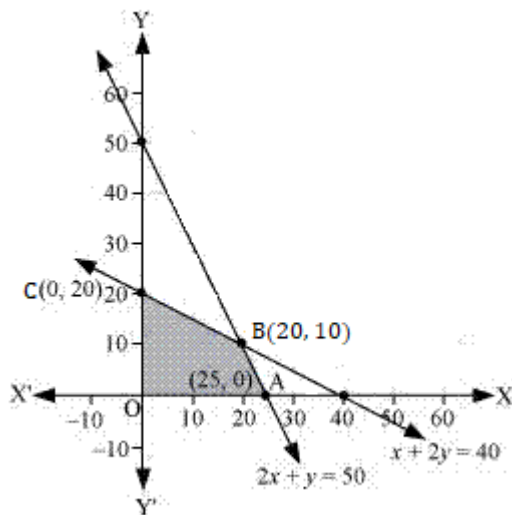
Subject to constraints

$$2x + y \leq 50 \dots$$
 (ii)

$$x + 2y \leq 40 \dots$$
 (iii)

$$x, y \geq 0 \dots$$
 (iv)

The feasible region is as follows:



The corner points are $A(25, 0)$, $B(20, 10)$, $C(0, 20)$ and $O(0, 0)$.

$$Z = x + y$$

$$\text{At } A(25, 0), Z = 25$$

$$\text{At } B(20, 10), Z = 30$$

$$\text{At } C(0, 20), Z = 20$$

$$\text{At } O(0, 0), Z = 0$$

Thus the maximum number of chocolates that can be made is 30 (20 of one kind and 10 of other kind).

37. Each student has the same chance of being chosen.

$$\text{Therefore, the probability of each student to be selected} = \frac{1}{15}$$



Now, the given information is as follows:

X	14	15	16	17	18	19	20	21
f	2	1	2	3	1	2	3	1

$$P(X = 14) = \frac{2}{15}, P(X = 15) = \frac{1}{15}, P(X = 16) = \frac{2}{15}, P(X = 17) = \frac{3}{15},$$

$$P(X = 18) = \frac{1}{15}, P(X = 19) = \frac{2}{15}, P(X = 20) = \frac{3}{15}, P(X = 21) = \frac{1}{15}$$

Thus, the probability distribution of random variable X is as follows:

X	14	15	16	17	18	19	20	21
P	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

$$\text{Mean of } X = E(X) = \sum x_i P(x_i)$$

$$= 14 \times \frac{2}{15} + 15 \times \frac{1}{15} + 16 \times \frac{2}{15} + 17 \times \frac{3}{15} + 18 \times \frac{1}{15} + 19 \times \frac{2}{15} + 20 \times \frac{3}{15} + 21 \times \frac{1}{15}$$

$$= \frac{1}{15}(28 + 15 + 32 + 51 + 18 + 38 + 60 + 21)$$

$$= \frac{263}{15}$$

$$E(X^2) = \sum x_i^2 P(x_i)$$

$$= 14^2 \times \frac{2}{15} + 15^2 \times \frac{1}{15} + 16^2 \times \frac{2}{15} + 17^2 \times \frac{3}{15} + 18^2 \times \frac{1}{15} + 19^2 \times \frac{2}{15} + 20^2 \times \frac{3}{15} + 21^2 \times \frac{1}{15}$$

$$= \frac{1}{15}(392 + 225 + 512 + 867 + 324 + 722 + 1200 + 441)$$

$$= \frac{4683}{15}$$

$$= 312.2$$

$$\text{Variance}(X) = E(X^2) - [E(X)]^2$$

$$= 312.2 - \left(\frac{263}{15}\right)^2$$

$$= 312.2 - 307.4177$$

$$= 4.7823$$

$$\approx 4.78$$

OR

Let E_1 , E_2 and E_3 be the respective events representing a scooter driver, a car driver and a truck driver.

Let A be the event that the person meets with an accident.

Total number of drivers = 2000 + 4000 + 6000 = 12000

$$P(E_1) = P(\text{Driver is a scooter driver}) = \frac{2000}{12000} = \frac{1}{6}$$

$$P(E_2) = P(\text{Driver is a car driver}) = \frac{4000}{12000} = \frac{1}{3}$$

$$P(E_3) = P(\text{Driver is a truck driver}) = \frac{6000}{12000} = \frac{1}{2}$$

$$P(A|E_1) = P(\text{Scooter driver met with an accident}) = 0.01 = \frac{1}{100}$$

$$P(A|E_2) = P(\text{Car driver met with an accident}) = 0.03 = \frac{3}{100}$$

$$P(A|E_3) = P(\text{Truck driver met with an accident}) = 0.15 = \frac{15}{100}$$

The probability that the driver is a scooter driver, given that he met with an accident is given by $P(E_1|A)$

Using Bayes' theorem, we get

$$\begin{aligned} P(E_1|A) &= \frac{P(E_1) \cdot P(A|E_1)}{P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2) + P(E_3) \cdot P(A|E_3)} \\ &= \frac{\frac{1}{6} \cdot \frac{1}{100}}{\frac{1}{6} \cdot \frac{1}{100} + \frac{1}{3} \cdot \frac{3}{100} + \frac{1}{2} \cdot \frac{15}{100}} \\ &= \frac{\frac{1}{6}}{\frac{1}{6} + \frac{1}{1} + \frac{15}{4}} \\ &= \frac{1}{52} \end{aligned}$$